(Pages : 4) **5663**

Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, December 2012
(2008 Scheme)
08.301 : ENGINEERING MATHEMATICS – II (CMPUNERFHBTA)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Evaluate $\iint_A xy\,dx\,dy$ where A is the domain bounded by x-axis, the ordinate $x = 2a$ and the curve $x^2 = 4ay$.

2. Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dx\,dy\,dz$

3. Write a condition for a line integral $\int_A^B \bar{F}.d\bar{r}$ is independent of the path joining the points A and B. Show that $\int_A^B [(2xy + z^3)dx + x^2dy + 3xz^2dz]$ is independent of the path joining A and B.

4. Obtain a Fourier series for $f(t) = 1 - t^2, -1 \leq t \leq 1$.

5. If S is any closed surface enclosing a volume V and $\bar{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$, prove that $\iint_S \bar{F}.\hat{n}ds = (a + b + c)V$.

6. Prove that for all values of x between $-\pi$ and π

$$\frac{1}{2}x = \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + ...$$

P.T.O.

7. Find the Fourier cosine transform of $f(x) = e^{-x}$.

8. Form the partial differential equation by eliminating the arbitrary constants a and b from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

9. Solve $z = xy\, p^2 q^2$.

10. Find a particular integral of $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$.

PART – B

Answer **one** full question from **each** Module. **Each** question carries **20** marks.

MODULE – I

11. a) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same. 6

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and $z = 0$. 7

c) Apply Stoke's theorem to evaluate $\int_C [(x+y)dx + (2x-z)\,dy + (y+z)\,dz]$ where C is the boundary of the triangle with vertices (0, 0, 0), (2, 0, 0) and (0, 3, 0). 7

12. a) By double integration, show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. 6

b) Using Green's theorem, evaluate $\oint_C \overline{F}.d\overline{r}$ counter clockwise around the boundary of C where $\overline{F} = \sin y\,\hat{i} + \cos x\,\hat{j}$ and C is the triangle with vertices (0, 0), $(\pi, 0)$ and $(\pi, 1)$. 7

c) If $\overline{F} = x\hat{i} + z\hat{j} + y\hat{k}$, evaluate $\iint_S \overline{F}.\hat{n}\,ds$ by divergence theorem where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$. 7

MODULE – II

13. a) If a Fourier series of e^{ax} in the interval $-\pi < x < \pi$ is

$$e^{ax} = \frac{2\sinh a\pi}{\pi}\left[\frac{1}{2a} - a\left(\frac{\cos x}{a^2+1} - \frac{\cos 2x}{a^2+2^2} + \frac{\cos 3x}{a^2+3^2} - \ldots\right) - \left(\frac{\sin x}{1^2+a^2} - \frac{2\sin 2x}{2^2+a^2} + \frac{3\sin 3x}{3^2+a^2} - \ldots\right)\right]$$

Show that $\cosh ax = \frac{2a}{\pi}\sinh a\pi\left[\frac{1}{2a^2} + \sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2+a^2}\right]$. 6

b) Obtain a Fourier series expansion for the function f(x) defined as

$$f(x) = \begin{cases} \pi x & \text{if } 0 \le x \le 1 \\ \pi(2-x) & \text{if } 1 \le x \le 2 \end{cases}$$ 7

c) Using the Fourier integral representation, show that

$$\int_0^{\infty}\frac{\omega\sin x\omega}{1+\omega^2}\,d\omega = \frac{\pi}{2}e^{-x}\,(x > 0).$$ 7

14. a) Obtain a Fourier Series expansion for $\sqrt{1-\cos x}$ in the interval $-\pi < x < \pi$. 6

b) Find the Fourier cosine transform of $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$. 7

c) Find the Fourier sine transform of $f(x) = \begin{cases} e^{-x}, & 0 \le x \le b \\ 0, & x > b \end{cases}$. 7

MODULE – III

15. a) Solve $(mz - ny)\,p + (nx - lz)\,q = ly - mx$. 6

b) Solve $4r - 4s + t = 16\log(x+2y)$ where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$ and $t = \frac{\partial^2 z}{\partial y^2}$. 7

c) Derive the one dimensional heat equation in the form $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$. 7

16. a) Solve $(D^2 - D'^2)\, z = \sin 2x$. 5

b) By method of separation of variables, solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given $u = 3e^{-y}$ when $x = 0$. 5

c) A tightly stretched, string of length 20 cm fastened at both ends is displaced from its equilibrium, by impartius to each of its points an initial velocity given by $\mu x(20 - x)$ where μ is a constant. Determine the displacement at a distance x from one end at any subsequent time t. 10
